

	Standard Deviation													
1	<p>The heights, in centimetres, of 7 netball players are given below.</p> <p style="text-align: center;">173, 176, 168, 166, 170, 180, 171</p> <p>Calculate the mean and standard deviation for these heights.</p>	4												
2	<p>In a bakery a sample of six fruit loaves is selected and their weights in grams are recorded.</p> <p style="text-align: center;">395 400 408 390 405 402</p> <p>Calculate the mean and standard deviation for these weights.</p> <p>New methods are introduced to ensure more consistent weights. A second sample is weighed where the mean is 400 grams and the standard deviation is 5.8 grams.</p> <p>Have the new methods been successful?</p>	4 1												
3	<p>The results for a group of students who sat tests in Physics and Chemistry are shown below</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">Physics (%)</td> <td style="padding: 5px;">10</td> <td style="padding: 5px;">18</td> <td style="padding: 5px;">26</td> <td style="padding: 5px;">32</td> <td style="padding: 5px;">49</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Chemistry (%)</td> <td style="padding: 5px;">25</td> <td style="padding: 5px;">35</td> <td style="padding: 5px;">30</td> <td style="padding: 5px;">40</td> <td style="padding: 5px;">41</td> </tr> </table> <p>(a) Calculate the mean and standard deviation for the physics test.</p> <p>(b) In the chemistry test the standard deviation was 6.8%. Make an appropriate comment on the distribution of marks in the two tests.</p>	Physics (%)	10	18	26	32	49	Chemistry (%)	25	35	30	40	41	4 1
Physics (%)	10	18	26	32	49									
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4	<p>A rugby team scored the following points in a series of seven matches</p> <p style="text-align: center;">13, 7, 0, 9, 7, 8, 5</p> <p>(a) Calculate the mean and standard deviation for this data sample.</p> <p>The team appoint a new coach and their next season produces a mean of 27 and a standard deviation of 3.25.</p> <p>(b) Make two comparisons about the performance of the team under the new coach.</p>	4 2												

5	<p>A new central heating system is installed in a house. Sample temperatures in degrees Celsius are recorded below.</p> <p style="text-align: center;">19 21 23 21 19 20</p> <p>(c) Calculate the mean and standard deviation for these temperatures.</p> <p>(d) For the greenhouse to be operating effectively the target temperature should be $20 \pm 0.6^{\circ}\text{C}$ and the standard deviation should be less than 2°C. Is this greenhouse operating effectively?</p>	<p style="text-align: center;">4</p> <p style="text-align: center;">2</p>
26 marks		

	Standard Deviation - Answers	26																																													
1	<p>Mark 1 Find the mean $\bar{x} = \frac{1204}{7} = 172$</p> <p>Mark 2 Complete the table of values for either formula</p> <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr><th>x</th><th>$x - \bar{x}$</th><th>$(x - \bar{x})^2$</th></tr> </thead> <tbody> <tr><td>173</td><td>1</td><td>1</td></tr> <tr><td>176</td><td>4</td><td>16</td></tr> <tr><td>168</td><td>-4</td><td>16</td></tr> <tr><td>166</td><td>-6</td><td>36</td></tr> <tr><td>170</td><td>-2</td><td>4</td></tr> <tr><td>180</td><td>8</td><td>64</td></tr> <tr><td>171</td><td>-1</td><td>1</td></tr> <tr> <td>$\sum x$ = 1204</td> <td>$\sum (x - \bar{x})$ = 0</td> <td>$\sum (x - \bar{x})^2$ = 138</td> </tr> </tbody> </table> <table border="1" style="display: inline-table;"> <thead> <tr><th>x</th><th>x^2</th></tr> </thead> <tbody> <tr><td>173</td><td>29929</td></tr> <tr><td>176</td><td>30976</td></tr> <tr><td>168</td><td>28224</td></tr> <tr><td>166</td><td>27556</td></tr> <tr><td>170</td><td>28900</td></tr> <tr><td>180</td><td>32400</td></tr> <tr><td>171</td><td>29241</td></tr> <tr> <td>$\sum x = 1204$</td> <td>$\sum x^2 = 207226$</td> </tr> </tbody> </table> <p>Mark 3 Substitute into the formulae $s = \sqrt{\frac{138}{6}}$ $s = \sqrt{\frac{207226 - \frac{1204^2}{7}}{6}}$</p> <p>Mark 4 Calculate the standard deviation $s = \sqrt{23} = 4.8$</p>	x	$x - \bar{x}$	$(x - \bar{x})^2$	173	1	1	176	4	16	168	-4	16	166	-6	36	170	-2	4	180	8	64	171	-1	1	$\sum x$ = 1204	$\sum (x - \bar{x})$ = 0	$\sum (x - \bar{x})^2$ = 138	x	x^2	173	29929	176	30976	168	28224	166	27556	170	28900	180	32400	171	29241	$\sum x = 1204$	$\sum x^2 = 207226$	4
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5

Mark 1 Find the mean $\bar{x} = \frac{123}{6} = 20.5$

Mark 2 Complete the table of values for either formula

x	$x - \bar{x}$	$(x - \bar{x})^2$
19	-1.5	2.25
21	0.5	0.25
23	2.5	6.25
21	0.5	0.25
19	-1.5	2.25
20	-0.5	0.25
$\sum x = 123$	$\sum (x - \bar{x}) = 0$	$\sum (x - \bar{x})^2 = 11.5$

x	x^2
19	361
21	441
23	529
21	441
19	361
20	400
$\sum x = 123$	$\sum x^2 = 2533$

Mark 3 Substitute into the formulae $s = \sqrt{\frac{11.5}{5}}$

$$s = \sqrt{\frac{2533 - \frac{123^2}{6}}{5}}$$

Mark 4 Calculate the standard deviation $s = 1.516 \dots$

Mark 5 Statement about the mean

The mean temperature lies within the target range which is $19.4^\circ\text{C} < 20.5^\circ\text{C} < 20.6^\circ\text{C}$

Mark 6 Statement about the standard deviation

The standard deviation is less than 2°C so the greenhouse **is operating successfully.**

6

